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Time complexity of recursive methods for computing the reliability of strict consecutive–k–out–of–n:F systems

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Abstract

A method is given for calculating the failure probability function for circular consecutive–k–out–of–n:F systems which operate in such a way that isolated strings of failures of length less than k (which do not cause system failure) do not occur, or are immediately corrected; ie, when system failure occurs it is because all failures present are in strings of length at least k. This paper estimate the time complexity of Papastavidis's recursive formula for computing the linear system failure probability and gives a recursive method to compute the circular consecutive–k–out–of–n:F systems

Keywords: Strict circular consecutive–k–out–of–n:F system; System reliability; Recursive method.

1. Introduction

Some methods of calculating the failure probability for consecutive–k–out–of–n:F systems, both recursive methods [1,4–7] and direct methods [8], do not rule out the situation that the failure mode (at least one string of k or more consecutive failures) is also accompanied by any possible strings of isolated failures of length less than k. For example, let $n=7$, $k=2$, F designate a failed component, and G designate a operational component; then the state $FGFGGFF$ is one of system failure only because of the last two F's, but there are also two isolated failure strings of length less than two. It seems reasonable to suppose, however, that in at least some applications of these systems, as might be the case with communication relay systems, isolated failure strings of length less than k –which may degrade performance but do not cause system failure–are, or can be, detected and corrected within an interval short enough that the normal operating mode can be considered to have no failed components. That is, Bollinger assumes here that although prevention of loss of system continuity is important enough that a consecutive–k–out–of–n:F design is used for protection, the detection and repair or replacement of isolated failed components occur quickly enough that the context is not that of the ordinary consecutive–k–out–of–n:F system [2]. In such a case, system failure will occur when and only when k or more consecutive components fail, *and* without any isolated failure strings of fewer than k consecutive components. Bollinger called such system *strict* consecutive–k–out–of–n:F systems, and in what follows their failure probability was calculated [2].

For example, in a consecutive–2–out–of–7: F system; then the following constitute failures of the linear system:

FFGGGGG

FFGGGFF

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GGFFFGG

GFFFFFG

FFFGFFF .

While the following configurations would cause failure of the linear system, they are assumed *not to occur* because they contain isolated failure strings of length less than 2:

FGFFFFF

FFGFFGF

FFFGGFG .

The following represents a system in working order:

GGGGGGG .

As might be anticipated, the failure probability for an ordinary system is extremely conservative compared to that for a strict system, and when the strict system applies, it might be possible to use the information this provides in design economies. Bollinger studies linear system with equal component probabilities. He presents a formula for calculating the system failure probability [2].

Papastavridis studies linear system with un-equal component probabilities [3]. He provides a recursive formula for computing the system failure probability. Furthermore, he gave a simple exact formula for the failure probability of the system with components of equal probability, ie, the i.i.d. case.

We distinguish linear and circular forms of strict consecutive–k–out–of–n :F systems that are analogous to consecutive–k–out–of–n :F systems.

This paper has two purposes. One is to estimate the time complexity of Papastavidis's recursive formula for computing the linear system failure probability. Another is to introduce a method to generate a computer enumeration for the reliability function of the strict circular consecutive–k–out–of–n :F system. The method computes the reliability in $O(nk^2)$ time, for the general case of un-equal component probabilities.

2. Notation & Assumptions

Notation

n number of components in the system.
k minimum number of consecutive components which cause system failure, $2 \le k \le n$
p_i operational probability of ith component.
q_i $\equiv 1.0 - p_i$, failure probability of ith component.
f first operational component.
t last operational component.
$F_L(i;j)$
failure probability (unreliability) of the linear system with components $i, i+1, \ldots, j$.
$F_C(n)$
failure probability (unreliability) of the circular system with n components.

Assumptions

A. Each component and system is either operational or failed; the probabilities of component failure are known.

B. For given k and n, all components are mutually statistically independent with unequal probabilities.

C. The system fails if and only if at least k consecutive components fail, and without any isolated failure strings of length less than k..

3. Linear system

Papastavridis studied the reliability of strict linear consecutive–k–out–of–n :F system where all the components have unequal failure probabilities . He has presented an equation to evaluate the failure probability (unreliability) of the strict consecutive–k–out– of–n :F system as follows [2].

Let $F_L(1;n:n_G)$ and $F_L(1;n:n_F)$ represent the failure probability of the system with component n being operational or failure, respectively. Hence

$$F_L(1;n) = F_L(1;n:n_G) + F_L(1;n:n_F) \quad \text{for all } n > k. \tag{1}$$

The system fails with the last component (component n) being operational if and only if the system fails at $(n-1)$ stages; hence

$$F_L(1;n:n_G) = p_n F_L(1;n-1) \tag{2}$$

Denote $F_L^*(1;i) = \prod_{j=1}^{i} p_j + F_L(1;i)$, for all $i \geq 1$, and $F_L^*(0;0) = 1.0$.

The same argument results in:

$$F_L(1;n:n_F) = q_n F_L(1;n-1:n-1_F) + p_{n-k} \cdot \prod_{i=n-k+1}^{n} q_i F_L^*(1;n-k-1) \tag{3}$$

The recursive equations (1)–(3) give:

$$F_L(1;n) = F_L(1;n:n_G) + F_L(1;n:n_F)$$

$$= p_n F_L(1;n-1) + q_n F_L(1;n-1:n-1_F) + p_{n-k} \prod_{i=n-k+1}^{n} q_i F_L^*(1;n-k-1)$$

$$= F_L(1;n-1) - q_n (F_L(1;n-1) - F_L(1;n-1;n-1_F) + p_{n-k} \prod_{i=n-k+1}^{n} q_i F_L^*(1;n-k-1)$$

$$= F_L(1;n-1) - p_{n-1} q_n F_L(1;n-2) + p_{n-k} \prod_{i=n-k+1}^{n} q_i F_L^*(1;n-k-1) \tag{4}$$

For fixed $k \geq 2$ and for $n \geq k+1$, the recursive formula is:

$$F_L(1;n) = F_L(1;n-1) - p_{n-1} q_n F_L(1;n-2) + p_{n-k} \cdot \prod_{i=n-k+1}^{n} q_i \cdot F_L(1;n-k-1) + \prod_{i=1}^{n-k} p_i \cdot \prod_{j=n-k+1}^{n} q_j . \tag{5}$$

The initial conditions are:

$$F_L(1:i) = 0.0, \text{ for } i = 0,1,2,\ldots,k-1 \text{ and } F_L(1;k) = q_1 q_2 \ldots q_k .$$

In the equation (5), first compute $Q(i)=\prod_{j=i-k+1}^{i} q_j$, for $i=2k,2k+1,\ldots,n$.

This requires $O(n+k)=O(n)$ time by first computing $O(k)$ and then computing $Q(i+1)=Q(i)q_{i+1}/q_{i-k+1}$ for each $i=2k,2k+1,\ldots,n-1$. Once this is done, then each $F_L(1;n)$ can be computed in constant time. Since there are n such $F_L(1;n)$'s to compute, we need another $O(n)$ time. Therefore the total time required is $O(n)$.

4. Circular system

Consider that the n components lie on a cycle. Suppose that the n components are labeled by the set $\{1,2,\ldots,n\}$ in a clockwise rotation (component n followed by component 1).

The component n has two states, operational or failed. So we have 2 events that the system is operational as follows:

A. If the component n is operational, the unreliability function is given by:

$$F_{CA}(n)=p_n\cdot F_L(1;n-1) \tag{6}$$

B. If the $l\,(k\le l\le n)$ consecutive components containing of the component n are failed, these components cause the system failure.

For $n\ge k+1$, such a pair (f,t) must exist for the system to fail. Then the unreliability function is given by:

$$F_{CB}(n)=\sum_{k\le f-1+n-t\le n}(p_f\prod_{i=1}^{f-1}q_i)\cdot(p_t\prod_{j=t+1}^{n}q_j)\cdot\{1.0-F_L(f+1;t-1)\}$$
$$+\sum_{k\le f-1+n-t\le n}\prod_{m=f}^{t}p_m\cdot\prod_{i=1}^{f-1}q_i\cdot\prod_{j=t+1}^{n}q_j \tag{7}$$

The initial conditions are: $R_{CB}(k)=q_1q_2\ldots q_k$ and $R_{CB}(i)=0.0$, for $i=0,1,\ldots,k-1$.

Furthermore, equation (7) is rewritten as follows:

$$F_{CB}(n)=\sum_{s=k}^{n-1}\sum_{i=0}^{s-1}p_{i+1}\cdot p_{n-s+i}\cdot\prod_{j=1}^{i}q_j\cdot\prod_{l=n-s+i+1}^{n}q_l\cdot\{1.0-F_L(i+2;n-s+i-1)\}$$
$$+\sum_{s=k}^{n-1}\sum_{i=0}^{s-1}\prod_{m=i+1}^{n-s+i}p_m\cdot\prod_{j=1}^{i}q_j\cdot\prod_{l=n-s+i+1}^{n}q_l \tag{8}$$

The events, A and B, are disjoint each other. Then the system unreliability, $F_C(n)$, can be written as:

$$F_C(n)=F_{CA}(n)+F_{CB}(n) \tag{9}$$

Next consider the time complexity of equation (9). The equation (6) is computed in $O(n)$ time. In equation (8), there are at most k^2 distinct products:

$$\sum_{s=k}^{n-1}\sum_{i=0}^{s-1}p_{i+1}\cdot p_{n-s+i}\cdot\prod_{j=1}^{i}q_j\cdot\prod_{l=n-s+i+1}^{n}q_l$$

which can be computed in $O(k^2)$. Furthermore, the linear system can be computed in $O(n)$ time as shown before. Since there are at most k^2 values of linear system to compute, the total time required is $O(k^2)+O(nk^2)=O(nk^2)$.

5. Conclusion

This paper estimated the time complexity of Papastavidis's recursive formula for computing the linear system failure probability and described a method to generate a computer enumeration for the reliability function of the strict circular consecutive–k–out–of–n :F system. The proposed method computes the reliability in $O(nk^2)$ time, for the general case of unequal component probabilities.

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