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Tables of Coefficients in Approximate Expressions for Integral Quantities of the Generalized TMA and Thornton Spectra

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This paper presents a summary of the coefficients in approximate expression for each of the integral quantities of a universal spectrum of wind waves in finite-depth water, which is a corrected version of either a generalized TMA (G-TMA) spectrum or a generalized Thornton (G-TMA) spectrum for 6 combinations of spectral parameters. The range of relative error associated with the approximate expression of spectrum-integrated quantity is given for each spectral condition. The tables may be very useful for an accurate and quick estimation of the spectrum-integrated quantities.

Key Words : generalized TMA and Thornton spectra, integral quantities, approximate expressions, table of coefficients

1. Introduction

Yamaguchi et al.[1] re-proposed a corrected form to the G-TMA spectrum proposed by Yamaguchi[2] for a universal frequency spectrum of wind waves in finite-depth water which has the maximum value at the supposed peak frequency. The correction was achieved by analytically introducing a non-trivial term associated with the finite-depth effect. They provided rather accurate approximate expressions for the integral quantities of the G-TMA and G-Thornton spectra with 6 combinations of 4 parameters related to the spectral form. Also they made a similar proposal to a G-Thornton spectrum and its spectrum-based integral quantities. But only a few coefficients for the approximate expressions were given due to the limitation of the allowed number of pages in the paper.

The purpose of this study is to make it possible to apply these highly accurate approximations by presenting the coefficients in each case of the 6 spectral conditions.

2. Integral Quantities of Universal Spectrum in Finite-Depth Water

2.1 Generalized TMA and Thornton Spectra

Both the G-TMA spectrum and the G-Thornton spectrum are written by the same form of expression as follows.

$$E(f) = Af_*^{-m}\exp\left\{-\left(\frac{m}{n}\right)f_*^{-n}(1-Cor)\right\}\gamma^{\exp\left\{-\frac{(1-f_*)^2}{2\sigma^2}\right\}}\Phi(kh) \tag{1}$$

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$$A = \alpha_m (2\pi) g^{-3} u_*^5 (2\pi\mu)^{-m}, \quad \mu = f_p u_* / g, \quad f_* = f / f_p \tag{2}$$

, where $E(f)$ is the energy density spectrum, f the frequency, α_m the equilibrium constant, g the acceleration of gravity, Cor the correction term, f_p the peak frequency, γ the peak enhancement parameter, σ the peak width parameter($\sigma = \sigma_a$; $f < f_p$, $\sigma = \sigma_b$; $f > f_p$), u_* the friction velocity of winds, μ the dimensionless peak frequency, and k the wave number corresponding to the frequency f and the water depth h .

The G-TMA spectrum has the Cor term and the finite-depth effect term $\Phi(kh)$ which are indicated as

$$Cor = \frac{2k_p h}{m(\sinh 2k_p h + 2k_p h)} \left\{ (m-3) + \frac{4k_p h (\cosh 2k_p h + 1)}{\sinh 2k_p h + 2k_p h} \right\} \tag{3}$$

$$\Phi(kh) = \frac{(\tanh kh)^{(m-1)/2}}{1 + 2kh / \sinh 2kh} \tag{4}$$

, where k_p is the wave number corresponding to the peak frequency f_p and the water depth h . The G-Thornton spectrum has the two terms shown as

$$Cor = \frac{2(m-3)}{m} \cdot \frac{2k_p h}{\sinh 2k_p h + 2k_p h} \tag{5}$$

$$\Phi(kh) = (\tanh kh)^{m-3} \tag{6}$$

In deep water ($kh \to 0$, $k_p h \to 0$), the correction term Cor tends to zero and the finite-depth effect term $\Phi(kh)$ tends to 1 in both spectra. The resulting expression of Eq.(1) reduces to the generalized JONSWAP(G-JONSWAP) spectrum as

$$E(f) = A f_*^{-m} \exp\left\{ -\left(\frac{m}{n}\right) f_*^{-n} \right\} \gamma^{\exp\left\{ -\frac{(1-f_*)^2}{2\sigma^2} \right\}} \tag{7}$$

2.2 Definition of Integral Quantities

The k -th order moment M_k of the G-TMA spectrum or the G-Thornton spectrum is formally written as

$$M_k = \int_0^\infty f^k E(f) df = A \cdot f_p^{k+1} \left[\int_0^\infty f_*^{k-m} \exp\left\{ -\left(\frac{m}{n}\right) f_*^{-n} (1 - Cor) \right\} \gamma^{\exp\left\{ -\frac{(1-f_*)^2}{2\sigma^2} \right\}} \Phi(kh) df_* \right] = A \cdot f_p^{k+1} \widetilde{M}_k \tag{8}$$

, where $\widetilde{M}_k$ indicates the dimensionless integration term in the square bracket [ ]. The total wave energy M_0 and its dimensionless form $\widetilde{M}_0$ (or ε) becomes

$$M_0 = \int_0^\infty E(f) df = A \cdot f_p \widetilde{M}_0 \tag{9}$$

The integral quantities I_k defined by $\widetilde{M}_k / \widetilde{M}_0$ can then be expressed as

$$I_k = \frac{\widetilde{M}_k}{\widetilde{M}_0} = \frac{(M_k / A f_p^{k+1})}{(M_0 / A f_p)} = \frac{1}{f_p^k} \frac{\int_0^\infty f^k E(f) df}{\int_0^\infty E(f) df} = \frac{1}{f_p^k} (\bar{f}_k)^k = \left(\frac{\bar{f}_k}{f_p}\right)^k = \left(\frac{T_p}{\bar{T}_k}\right)^k \quad ; \ k = \pm 1, \ \pm 2 \tag{10}$$

, where $\bar{f}_k$ and $\bar{T}_k (= 1/\bar{f}_k)$ are the k -th order moment-based average frequency and its reciprocal (average period).

Yamaguchi et al.[1] made approximate expressions of the integral quantities for k =2, 1, 0, -1 -2 in 6 spectral conditions. These integral quantities are expressed as

$$I_2 = \widetilde{T}_{m02}^{-2} = \left(T_p/\overline{T}_2\right)^2 > 1,\ \ I_1 = \widetilde{T}_{m01}^{-1} = \left(T_p/\overline{T}_1\right) > 1,\ \ I_0 = \widetilde{M}_0 = \varepsilon,$$

$$I_{-1} = \widetilde{T}_{-01} = \left(T_p/\overline{T}_{-1}\right)^{-1} < 1,\ \ I_{-2} = \widetilde{T}_{-02}^{2} = \left(T_p/\overline{T}_{-2}\right)^{-2} < 1 \tag{11}$$

In the present study, new approximate expressions for two kinds of spectral width parameter are suggested. One is the spectral width parameter ν used by Longuet-Higgins[3], which is defined as

$$\nu^2 = \frac{M_0 \cdot M_2}{M_1^2} - 1 = \frac{\widetilde{M}_0 \widetilde{M}_2}{\widetilde{M}_1^2} - 1 \tag{12}$$

The other is the spectral peakedness parameter Q_p proposed by Goda[4], which is defined as

$$Q_p = 2\int_0^\infty f\{E(f)\}^2 df \bigg/ \left(\int_0^\infty E(f)df\right)^2 = 2\int_0^\infty f_* E(f_*)^2 df_* \bigg/ \widetilde{M}_0^2 \tag{13}$$

For the case of γ=1 in deep water ($kh \to \infty$, $k_p h \to \infty$), the integral quantities I_k and the spectral width parameters of ν and Q_p have the following analytical forms respectively.

$$I_k = \left(\frac{m}{n}\right)^{k/n} \Gamma\left(\frac{m-k-1}{n}\right) \bigg/ \Gamma\left(\frac{m-1}{n}\right) \tag{14}$$

$$I_0 = \varepsilon = \frac{1}{n}\left(\frac{m}{n}\right)^{-(m-1)/n} \Gamma\left(\frac{m-1}{n}\right) \tag{15}$$

$$\nu^2 = \frac{\Gamma[(m-1)/n] \cdot \Gamma[(m-3)/n]}{\{\Gamma[(m-2)/n]\}^2} \tag{16}$$

$$Q_p = \frac{2n}{2^{2(m-1)/n}} \frac{\Gamma[2(m-1)/n]}{\{\Gamma[(m-1)/n]\}^2} \tag{17}$$

, where Γ is the gamma function. In the case of $m = 5$ and $n = 4$, the analytical integrals with 5 significant digits becomes

$$\widetilde{T}_{m02}^{-2} = 1.9817,\ \ \widetilde{T}_{m01}^{-1} = 1.2957,\ \ \varepsilon = 0.2,\ \ \widetilde{T}_{-01} = 0.85722,\ \ \widetilde{T}_{-02}^{2} = 0.79267,\ \ \nu = 0.42467,\ \ Q_p = 2 \tag{18}$$

2.3 Data Sample of Integral Quantities

Under the fixed conditions of the parameters m, n, σ_a and σ_b, each combination corresponding to TMA, FRF and another spectrum, the numerical integrations of the dimensionless k-th order spectral moments $\widetilde{M}_k$ (k=2, 1, 0, -1, -2) and the dimensionless first-order moment of squared spectrum in the Q_p expression are carried out for a range of γ from 1 to 10(1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 6, 7, 8, 9, 10; 14 cases) for both deep and finite-depth water, and a range of $T_p\sqrt{g/h}$ from 1 to 10(with an increment of 0.1) and from 11 to 50 (with an increment of 1) in finite-depth water (131 cases). The total number of runs in finite-depth water is 14×131=1,834.The final estimates are obtained by adding the analytical integration from a high frequency to infinity. The integral quantities I_k and the spectral width parameters of ν and Q_p are obtained using the computed dimensionless spectral moments.

3. Approximate Expressions for Integral Quantities and Spectral Width Parameters and Their Coefficients

3.1 Approximation for Integral Quantities and Spectral Width Parameters

Under the fixed combination of the spectral parameters m, n, σ_a and σ_b, the integral quantities and the spectral width parameters depend in deep water only on γ and in finite-depth water on both γ and h/L_p (or $k_p h$), where $L_p = 2\pi/k_p$. In deep water, the following expression proposed by Yamaguchi[5] is used for analytically approximating the numerically computed integral quantities $(I_k)_{deep}$.

$$(I_k)_{deep} = a\gamma^b + (I_{k0})_{deep} - a \tag{19}$$

Eq.(19) gives the exact value of $(I_{k0})_{deep}$ at $\gamma = 1$. The coefficients a , b in Eq.(19) are determined with the least squares-errors method. For the cases of spectral width parameters, a 3-point-fixed 4th-order polynomial is applied to improve a slightly lower fitting ability of Eq.(19). The polynomial for the spectral width parameter ν_{deep} is written as

$$\nu_{deep} = a\gamma^4 + b\gamma^3 + c\gamma^2 + d\gamma + e \tag{20}$$

Imposing the conditions that the polynomial passes through each of the 3 data points prescribed at $\gamma = 1$, $\gamma = 10$ and a value of γ between 1 and 10, reduces the number of the coefficients to be determined to 2. The coefficients are determined by using the least-squares method.

In finite-depth water, the following expression with 6 unknown parameters (p , q , r , s , u , w) is fitted to the data sample of $I_k/(I_k)_{deep}$ normalized with $(I_k)_{deep}$ which indicates the finite-depth water effect.

$$\frac{I_k}{(I_k)_{deep}} = \left\{1 + \frac{r(k_p h)^u}{\sinh r(k_p h)^u}\right\}^p \cdot \left\{\tanh s(k_p h)^w\right\}^q , \quad k_p h = 2\pi h/L_p \tag{21}$$

The right-hand-side term of Eq.(21) reduces to 1 with increasing $k_p h$. The same form of expression is used for the spectral width parameter ν or Q_p . The least-squares-method solution of the 3 parameters (p , q , r) is obtained from the log-transformed Eq.(21) under prescribed combining for the remaining 3 parameters(s , u , w) and then a quasi-optimum solution for the 6 parameters is selected from the solutions of the parameters(p , q , r) for a huge number of combinations of the prescribed parameters(s , u , w) in the sense of least-squares-errors.

Each of the parameters (p , q , r) for an individual integral quantity is approximated using the same form as Eq.(19). For instance, it is written for the parameter p as

$$p = a_p\gamma^{b_p} + p_0 - a_p \ ; \ p_0 = p(\gamma = 1) \tag{22}$$

, where p_0 is p at γ =1 respectively. In the case of a slightly lower fitting ability of Eq.(22), a 3-point-fixed 4th-order polynomial is used as well as in the case in deep water conditions.

$$p = a_p\gamma^4 + b_p\gamma^3 + c_p\gamma^2 + d_p\gamma + e_p \tag{23}$$

The computation method for determining the parameters in Eq.(22) or Eq.(23) is the same as the above-mentioned method.

The integral quantities in finite-depth water I_k are approximated by a multiplication of Eq.(21) with Eq.(19) as

$$I_k = \left\{a\gamma^b + (I_{k0})_{deep} - a\right\} \cdot \left\{1 + \frac{r(k_p h)^u}{\sinh r(k_p h)^u}\right\}^p \cdot \left\{\tanh s(k_p h)^w\right\}^q \tag{24}$$

, where the parameter (a , b , $(I_{k0})_{deep}$, s , u , w) has a constant value and the parameter(p , q , r) is γ -dependent. The spectral width parameter ν in finite-depth water is approximated as follows.

$$\nu = \left(a\gamma^4 + b\gamma^3 + c\gamma^2 + d\gamma + e\right) \cdot \left\{1 + \frac{r\left(k_p h\right)^u}{\sinh r\left(k_p h\right)^u}\right\}^p \cdot \left\{\tanh s\left(k_p h\right)^w\right\}^q \tag{25}$$

Each of the parameters (p , q , r) is expressed by either Eq.(22) or Eq.(23). The spectral peakedness parameter Q_p in finite-depth water is also expressed by the same type of expression as Eq.(25).

3.2 Coefficients in Approximate Expression

Table 1 gives the 6 spectral combinations in finite-depth water used in the derivation. The combinations include not only the new versions of TMA spectrum (Case 1), FRF spectrum (Case 2) and Thornton-type spectrum (Case 6) but also the original TMA spectrum (Case 5). Tables 2 - 7 give coefficients in the approximate expression for each of the integral quantities and spectral width parameters under the 6 spectral conditions. The first line in a section for each integral quantity or spectral width parameter stands for the coefficients of the approximate expression in deep water and the fixed coefficients (s , u , w) of Eq.(21) in finite-depth water. The second to fourth lines give the coefficients of the approximate expression in finite-depth water. Each approximate expression for the parameter p or q or r is due to either Eq.(22) with the 3 coefficients of a , b and y_0=(p_0 , q_0 , r_0) or Eq.(23) with the 5 coefficients of a , b , c , d and e .

Table 1 Spectral conditions used in the study.

Case	m	n	σ_a	σ_b	C_{or}	Remarks
1	5	4	0.07	0.09	$C_{or}\neq 0$	Corrected TMA spectrum(1)
2	4	4	0.115	0.115	$C_{or}\neq 0$	Corrected FRF spectrum
3	4.5	3.5	0.07	0.09	$C_{or}\neq 0$	Corrected Yama spectrum
4	5	4	0.2	0.2	$C_{or}\neq 0$	Corrected TMA spectrum(2)
5	5	4	0.07	0.09	$C_{or}=0$	Original TMA spectrum
6	5	4	0.07	0.09	$C_{or}\neq 0$	Corrected Thornton Spectrum

Table 8 summarizes the relative error in deep water δ_{deep} and that in finite-depth water δ . As can be observed in this table, the accuracy of each approximate expression is rather high. But the relative error δ tends to become greater in the cases of $\widetilde{T}_{m02}^{-2}$ and ν based on the higher-order moment and in the case of Q_p based on the first-order moment of the squared spectrum.

As an example, using the coefficients in Table 3 leads to the following approximate expression for dimensionless energy $\varepsilon \left(= \widetilde{M}_0\right)$ in the case of a corrected FRF spectrum (Case 2).

$$\varepsilon = \left\{-0.11356\gamma^{0.81197} + (0.30635 - 0.11356)\right\} \cdot \left\{1 + \frac{r\left(k_p h\right)}{\sinh r\left(k_p h\right)}\right\}^p \cdot \left\{\tanh 0.4\left(k_p h\right)^{1.8}\right\}^q$$

$$\begin{aligned} p &= 1.9153\gamma^{-0.10300} + (1.2846 - 1.9153) \\ q &= 0.44408\times10^{-5}\gamma^4 - 0.11938\times10^{-3}\gamma^3 + 0.11601\times10^{-2}\gamma^2 - 0.45757\times10^{-2}\gamma + 0.83320 \\ r &= -0.91892\gamma^{0.11729} + (1.9950 + 0.91892) \end{aligned} \tag{26}$$

The relative error δ ranges from -0.79 % to 0.54%, as given in Table 8. Fig. 1 shows a comparison between approximate expression for ε and the data of ε , and the relative error δ in the case of a corrected FRF spectrum. The curve fits to the data very well and the error is significantly small on a whole range of h/L_p .

Table 2 Coefficients in approximate expression for integral quantity of corrected G-TMA spectrum (m=5, n=4, σ_a=0.07, σ_b=0.09, $Cor \neq 0$; TMA spectrum(1)).

para.	a	b	y_0 or c	d	e	u	w	s
$\widetilde{T}_{m02}^{-2}$	5.2462	-0.053888	1.9817			0.3	2.1	0.5
p	0.062300	1.1555	-1.5657					
q	0.013748	0.89091	-0.30766					
r	-0.20787×10^{-3}	0.56524×10^{-2}	-0.058313	0.22906	4.6499			
$\widetilde{T}_{m01}^{-1}$	1.6416	-0.051081	1.2957			0.6	1.8	0.1
p	-0.16828	0.49600	0.90318					
q	-0.010264	0.42398	0.037314					
r	0.47944×10^{-4}	-0.13067×10^{-2}	0.012980	-0.053152	3.9523			
ε	0.065329	0.80152	0.20000			1.0	1.8	0.4
p	35.973	-0.47866×10^{-2}	1.8250					
q	0.69679×10^{-2}	-0.65156	1.1114					
r	-0.31404	0.17850	2.0659					
$\widetilde{T}_{-01}$	-0.77319	-0.051445	0.85722			1.0	1.0	0.3
p	0.33169	0.16489	-0.28577					
q	-0.052724	-0.11283	-0.018349					
r	0.19017	-0.45675	3.8284					
$\widetilde{T}_{-02}^{2}$	-1.0300	-0.056099	0.79267			1.0	1.0	0.3
p	1.1369	0.089485	-0.45951					
q	-0.096397	-0.13878	-0.039571					
r	0.22515	-0.38496	3.7972					
ν	-0.44634×10^{-6}	-0.16432×10^{-4}	0.91935×10^{-3}	-0.018876	0.44264	1.7	3.0	2.9
p	0.29037	0.27871	0.25356					
q	-0.12536×10^{-5}	0.32191×10^{-4}	-0.25987×10^{-3}	-0.23636×10^{-3}	-0.10106			
r	-0.61096×10^{-4}	0.14450×10^{-2}	-0.011245	0.026837	2.9148			
Q_p	0.35478×10^{-3}	-0.81871×10^{-2}	0.047527	0.39338	1.5669	1.0	0.5	0.4
p	0.23614×10^{-3}	-0.64326×10^{-2}	0.062289	-0.22076	-0.51193			
q	0.27535×10^{-4}	-0.74437×10^{-3}	0.73000×10^{-2}	-0.029803	0.010586			
r	0.44985	-1.1517	3.1261					

$y_0 = (I_{k0})_{deep}$, p_0, q_0, r_0

Table 3 Coefficients in approximate expression for integral quantity of corrected G-TMA spectrum (m=4, n=4, σ_a=0.115, σ_b=0.114, $Cor \neq 0$; FRF spectrum).

para.	a	b	y_0 or c	d	e	u	w	s
$\widetilde{T}_{m02}^{-2}$	4.9773	-0.13346	2.9587			1.7	2.8	0.1
p	-0.49539	-0.10515	-0.63188					
q	0.29217×10^{-2}	1.0500	-0.23138					
r	0.47394×10^{-4}	-0.12946×10^{-2}	0.012878	-0.047546	0.97602			
$\widetilde{T}_{m01}^{-1}$	1.1480	-0.13161	1.4464			0.2	2.6	0.4
p	0.57828×10^{-3}	-0.015245	0.14032	-0.42749	-1.7252			
q	0.011914	0.66206	-0.11245					
r	-0.40441×10^{-3}	0.010986	-0.11080	0.50023	5.8250			
ε	0.11356	0.81197	0.30635			1.0	1.8	0.4
p	1.9153	-0.10300	1.2846					
q	0.44408×10^{-5}	-0.11938×10^{-3}	0.11601×10^{-2}	-0.45757×10^{-2}	0.83320			
r	-0.91892	0.11729	1.9950					
$\widetilde{T}_{-01}$	-0.45923	-0.14071	0.81605			0.8	1.8	0.1
p	0.73375	0.13425	-0.46684					
q	-0.12634×10^{-5}	-0.48182×10^{-6}	-0.02281					
r	0.13356	-0.68428	3.6756					
$\widetilde{T}_{-02}^{2}$	-0.62177	-0.15444	0.73967			0.8	1.8	0.1
p	6.7481	0.028566	-0.75808					
q	-0.14174	-0.10166	-0.04565					
r	0.11728	-0.74072	3.6134					
ν	-0.10928×10^{-4}	0.25451×10^{-3}	-0.14055×10^{-2}	-0.016325	0.66108	1.1	2.7	0.3
p	-0.37470×10^{-4}	0.12636×10^{-2}	-0.017560	0.12470	-0.45164			
q	-0.15135×10^{-5}	0.53760×10^{-4}	-0.72750×10^{-3}	0.22967×10^{-2}	-0.12699			
r	-0.10305×10^{-3}	0.38531×10^{-2}	-0.034240	-0.11686	2.7926			
Q_p	0.25164×10^{-3}	-0.54325×10^{-2}	0.023515	0.37171	1.2793	0.9	1.2	0.1
p	0.36240×10^{-3}	-0.96511×10^{-2}	0.090721	-0.31584	-0.49351			
q	0.23286×10^{-4}	-0.63082×10^{-3}	0.61713×10^{-2}	-0.025030	0.012519			
r	0.63915	-1.0785	3.2905					

$y_0 = (I_{k0})_{deep}$, p_0, q_0, r_0

Table 4 Coefficients in approximate expression for integral quantity of corrected G-TMA spectrum (m=4.5, n=3.5, σ_a=0.07, σ_b=0.09, $Cor \neq 0$; Yama spectrum).

para.	a	b	y_0 or c	d	e	u	w	s
$\widetilde{T}_{m02}^{-2}$	-32.825	0.010830	2.3868			0.3	2.6	0.5
p	0.24117×10^{-3}	-0.64781×10^{-2}	0.062383	-0.16790	-1.0580			
q	0.12712×10^{-4}	-0.32702×10^{-3}	0.29217×10^{-2}	-0.35167×10^{-2}	-0.26497			
r	-0.32673×10^{-3}	0.84537×10^{-2}	-0.086653	0.36474	4.3848			
$\widetilde{T}_{m01}^{-1}$	-6.8517	0.013666	1.3710			0.2	2.7	0.4
p	0.29668×10^{-3}	-0.79708×10^{-2}	0.074903	-0.20058	-2.3264			
q	0.011220	0.61748	-0.10333					
r	-0.72670	-0.41947	6.3672					
ε	0.063957	0.79891	0.22222			1.0	1.8	0.4
p	-4.6772	0.032719	1.6222					
q	0.29458×10^{-5}	-0.82131×10^{-4}	0.85211×10^{-3}	-0.38562×10^{-2}	0.97134			
r	-0.32891	0.20817	1.9789					
$\widetilde{T}_{-01}$	2.9074	0.01385	0.83741			0.9	1.7	0.1
p	0.28596	0.22005	-0.37391					
q	0.15169	0.028470	-0.017405					
r	0.20117	-0.36677	3.8029					
$\widetilde{T}_{-02}^{2}$	5.8463	0.96717×10^{-2}	0.77148			0.9	1.8	0.1
p	0.96112	0.13008	-0.61925					
q	-0.35344	-0.028474	-0.036392					
r	0.23458	-0.29328	3.7794					
ν	-0.29873×10^{-5}	0.61742×10^{-4}	-0.99879×10^{-5}	-0.014578	0.53400	2.0	3.0	0.2
p	-2.1941	-0.022901	-0.23344					
q	-0.36956×10^{-2}	0.80219	-0.10302					
r	-1.3953	0.22010	1.4552					
Q_p	0.37599×10^{-3}	-0.91318×10^{-2}	0.63782×10^{-1}	0.26361	1.4314	0.9	1.2	0.1
p	0.25693×10^{-3}	-0.72585×10^{-2}	0.074273	-0.29307	-0.51237			
q	0.17022×10^{-4}	-0.48306×10^{-3}	0.50356×10^{-2}	-0.022010	0.43212×10^{-2}			
r	0.51320	-1.1350	3.2618					

$y_0 = (I_{k0})_{deep}$, p_0, q_0, r_0

Table 5 Coefficients in approximate expression for integral quantity of corrected G-TMA spectrum (m=5, n=4, σ_a=0.2, σ_b=0.2, $Cor \neq 0$; TMA spectrum(2)).

para.	a	b	y_0 or c	d	e	u	w	s
$\widetilde{T}_{m02}^{-2}$	1.1731	-0.43085	1.9817			0.3	2.2	0.5
p	0.30677×10^{-3}	-0.71492×10^{-2}	0.040022	0.15586	-1.7625			
q	0.13827×10^{-4}	-0.29396×10^{-3}	0.68852×10^{-3}	0.026564	-0.31921			
r	-0.17448×10^{-3}	0.66886×10^{-2}	-0.077299	0.18039	4.8407			
$\widetilde{T}_{m01}^{-1}$	0.35751	-0.41650	1.2957			0.2	2.5	0.4
p	1.9807	0.26628	-2.5894					
q	0.13102	0.16061	-0.09730					
r	-0.29054×10^{-3}	0.72406×10^{-2}	-0.062455	0.19464	6.4902			
ε	0.12098	0.85198	0.20000			1.0	1.8	0.4
p	0.73252	-0.35787	1.8250					
q	0.60185×10^{-5}	-0.14978×10^{-3}	0.12865×10^{-2}	-0.42014×10^{-2}	1.1145			
r	0.44977	-0.26782	2.0659					
$\widetilde{T}_{-01}$	-0.18096	-0.41041	0.85722			0.8	1.8	0.1
p	-0.78195	-0.17897	-0.37604					
q	-0.032055	-0.21949	-0.018710					
r	0.44217×10^{-4}	-0.10926×10^{-2}	0.92435×10^{-2}	-0.022246	3.6543			
$\widetilde{T}_{-02}^{2}$	-0.27424	-0.42194	0.79267			0.9	1.8	0.1
p	-0.83444	-0.26051	-0.52570					
q	-0.036455	-0.31856	-0.028188					
r	0.021516	0.92598	3.6851					
ν	0.80524×10^{-5}	-0.29978×10^{-3}	0.46911×10^{-2}	-0.043677	0.46395	1.3	2.4	3.0
p	-0.97770	-0.19447	0.33525					
q	-0.75861×10^{-5}	0.18902×10^{-3}	-0.16002×10^{-2}	0.44513×10^{-2}	-0.12283			
r	0.12215	-0.74605	3.4015					
Q_p	-0.25056×10^{-3}	0.77314×10^{-2}	-0.96834×10^{-1}	0.70316	1.3862	0.9	1.2	0.1
p	0.39593×10^{-3}	-0.95328×10^{-2}	0.075828	-0.18216	-0.62676			
q	0.21416×10^{-4}	-0.54287×10^{-3}	0.47889×10^{-2}	-0.016373	-0.11175×10^{-2}			
r	0.98486	-0.36225	3.0930					

$y_0 = (I_{k0})_{deep}$, p_0, q_0, r_0

Table 6 Coefficients in approximate expression for integral quantity of corrected G-TMA spectrum (m=5, n=4, σ_a=0.07, σ_b=0.09, $Cor = 0$; original TMA spectrum).

para.	a	b	y_0 or c	d	e	u	w	s
$\tilde{T}_{m02}^{-2}$	5.2462	-0.053888	1.9817			0.5	1.5	0.5
p	4.1763	-0.040479	1.5801					
q	0.36807×10^{-4}	-0.97657×10^{-3}	0.93464×10^{-2}	-0.037228	-0.065392			
r	0.15986×10^{-3}	-0.43357×10^{-2}	0.042906	-0.16710	4.1222			
$\tilde{T}_{m01}^{-1}$	1.6416	-0.051081	1.2957			0.5	2.0	2.0
p	-0.74575×10^{-4}	0.21039×10^{-2}	-0.021873	0.080176	0.97097			
q	-0.93785×10^{-5}	0.26893×10^{-3}	-0.29355×10^{-2}	0.014469	0.024384			
r	0.34112	0.30949	4.0097					
ε	0.065329	0.80152	0.20000			0.9	1.0	1.2
p	-2.5625	0.091147	-0.52441					
q	0.18394×10^{2}	-0.59802×10^{-3}	2.0070					
r	-0.56759	-0.32888	1.8716					
$\tilde{T}_{-01}$	-0.77319	-0.051445	0.85722			0.5	2.0	2.0
p	0.25752×10^{-4}	-0.62811×10^{-3}	0.42555×10^{-2}	0.021198	-0.73528			
q	0.75235×10^{-5}	-0.19612×10^{-3}	0.17971×10^{-2}	-0.53994×10^{-2}	-0.037600			
r	-1.6397	-0.093543	3.6276					
$\tilde{T}_{-02}^{2}$	-1.0300	-0.056099	0.79267			0.5	2.0	2.0
p	0.22063	0.51065	-1.1664					
q	0.22187×10^{-2}	1.0706	-0.069784					
r	-0.76934	-0.19832	3.4257					
ν	-0.44634×10^{-6}	-0.16432×10^{-4}	0.91935×10^{-3}	-0.018876	0.44264	0.8	2.0	0.9
p	0.43496	0.36582	-0.032315					
q	0.10841	0.082612	-0.16175					
r	0.41461×10^{-3}	-0.011045	0.10469	-0.40475	3.0747			
Q_p	0.35478×10^{-3}	-0.81871×10^{-2}	0.047527	0.39338	1.5669	1.0	2.0	2.0
p	0.78173×10^{-4}	-0.32390×10^{-2}	0.046608	-0.25274	-0.42662			
q	-0.62497×10^{-4}	0.14917×10^{-2}	-0.011969	0.035264	-0.022212			
r	1.0847	-2.1357	3.6476					

$y_0 = (I_{k0})_{deep}$, p_0, q_0, r_0

Table 7 Coefficients in approximate expression for integral quantity of corrected G-Thornton spectrum (m=5, n=4, σ_a=0.07, σ_b=0.09, $Cor \neq 0$; Thornton spectrum).

para.	a	b	y_0 or c	d	e	u	w	s
$\tilde{T}_{m02}^{-2}$	5.2462	-0.053888	1.9817			1.0	2.0	1.0
p	-0.20553×10^{-4}	0.53235×10^{-3}	-0.51856×10^{-2}	0.022238	-0.063637			
q	0.92784×10^{-2}	0.78772	-0.21479					
r	0.37919×10^{-3}	-0.90836×10^{-2}	0.088597	-0.33889	1.6252			
$\tilde{T}_{m01}^{-1}$	1.6416	-0.051081	1.2957			1.0	2.0	1.5
p	-0.035770	0.50879	0.17845					
q	0.31585×10^{-2}	0.70623	-0.043432					
r	0.75356	-0.39215	5.1665					
ε	0.065329	0.80152	0.20000			0.9	1.3	0.8
p	3.6144	-0.041154	1.7675					
q	0.38146×10^{-5}	-0.95274×10^{-4}	0.80579×10^{-3}	-0.17929×10^{-2}	1.5222			
r	-0.59554	0.11306	2.7552					
$\tilde{T}_{-01}$	-0.77319	-0.05145	0.85722			0.7	2.2	0.1
p	0.30688	0.19869	-0.33603					
q	0.063595	0.044655	-0.011494					
r	0.060223	-1.0937	4.5428					
$\tilde{T}_{-02}^{2}$	-1.0300	-0.056099	0.79267			0.7	2.3	0.1
p	0.95646	0.12009	-0.54447					
q	-0.25982	-0.024025	-0.022451					
r	0.077047	-0.79971	4.5003					
ν	-0.44634×10^{-6}	-0.16432×10^{-4}	0.91935×10^{-3}	-0.018876	0.44264	1.7	2.5	3.0
p	0.25171	0.24807	0.15175					
q	-0.62654×10^{-2}	0.60790	-0.11753					
r	-0.25260	-0.86755	3.6052					
Q_p	0.35478×10^{-3}	-0.81871×10^{-2}	0.047527	0.39338	1.5669	1.0	2.0	0.5
p	0.21267×10^{-3}	-0.56637×10^{-2}	0.053624	-0.19107	-0.29818			
q	-0.084467	0.057496	0.025002					
r	0.34667	-1.4416	4.1826					

$y_0 = (I_{k0})_{deep}$, p_0, q_0, r_0

Table 8 Range of relative error in approximate expression for integral quantity of corrected G-TMA and G-Thornton spectra.

G-TMA Case 1: m=5, n=4, σ_a=0.07, σ_b=0.09, $Cor \neq 0$		
para.	δ_{deep} %	δ %
$\widetilde{T}_{m02}^{-2}$	-0.39 ~ 0.25	-1.59 ~ 1.78
$\widetilde{T}_{m01}^{-1}$	-0.14 ~ 0.10	-0.93 ~ 0.56
ε	-0.24 ~ 0.11	-0.53 ~ 0.46
$\widetilde{T}_{-01}$	-0.06 ~ 0.10	-0.61~ 0.68
$\widetilde{T}_{-02}^{2}$	-0.09 ~ 0.15	-1.09 ~ 1.18
ν	-0.02 ~ 0.02	-0.98 ~ 2.72
Q_p	-0.42 ~ 0.36	-1.59 ~ 1.32

G-TMA Case 2 : m=4, n=4, σ_a=0.115, σ_b=0.114, $Cor \neq 0$		
para.	δ_{deep} %	δ %
$\widetilde{T}_{m02}^{-2}$	-0.70 ~ 0.41	-1.31 ~ 1.22
$\widetilde{T}_{m01}^{-1}$	-0.23 ~ 0.15	-1.00 ~ 0.94
ε	-0.24 ~ 0.11	-0.79 ~ 0.54
$\widetilde{T}_{-01}$	-0.09 ~ 0.16	-0.59 ~ 0.89
$\widetilde{T}_{-02}^{2}$	-0.14 ~ 0.25	-1.15 ~ 1.55
ν	-0.09 ~ 0.07	-1.17 ~ 1.14
Q_p	-0.42 ~ 0.39	-1.34 ~ 1.35

G-TMA Case 3 : m=4.5, n=3.5, σ_a=0.07, σ_b=0.09, $Cor \neq 0$		
para.	δ_{deep} %	δ %
$\widetilde{T}_{m02}^{-2}$	-0.48 ~ 0.30	-1.45 ~ 1.73
$\widetilde{T}_{m01}^{-1}$	-0.17 ~ 0.12	-0.98 ~ 0.55
ε	-0.22 ~ 0.10	-0.73 ~ 0.60
$\widetilde{T}_{-01}$	-0.07 ~ 0.11	-0.62 ~ 0.78
$\widetilde{T}_{-02}^{2}$	-0.10 ~ 0.16	-1.19 ~ 1.47
ν	-0.03 ~ 0.02	-1.30 ~ 1.89
Q_p	-0.45 ~ 0.36	-1.54 ~ 1.25

G-TMA Case 4 : m=5, n=4, σ_a=0.2, σ_b=0.2, $Cor \neq 0$		
para.	δ_{deep} %	δ %
$\widetilde{T}_{m02}^{-2}$	-0.34 ~ 0.22	-1.49 ~ 1.63
$\widetilde{T}_{m01}^{-1}$	-0.15 ~ 0.08	-0.79 ~ 0.37
ε	-0.27 ~ 0.09	-0.59 ~ 0.44
$\widetilde{T}_{-01}$	-0.05 ~ 0.11	-0.44 ~ 0.70
$\widetilde{T}_{-02}^{2}$	-0.08 ~ 0.18	-0.91 ~ 1.28
ν	-0.08 ~ 0.02	-1.01 ~ 3.01
Q_p	-0.08 ~ 0.10	-1.20 ~ 1.12

G-TMA Case 5 : m=5, n=4, σ_a=0.07, σ_b=0.09, $Cor = 0$		
para.	δ_{deep} %	δ %
$\widetilde{T}_{m02}^{-2}$	-0.39 ~ 0.25	-0.74 ~ 0.66
$\widetilde{T}_{m01}^{-1}$	-0.14 ~ 0.10	-0.46 ~ 0.26
ε	-0.24 ~ 0.11	-0.70 ~ 0.54
$\widetilde{T}_{-01}$	-0.06 ~ 0.10	-0.24 ~ 0.45
$\widetilde{T}_{-02}^{2}$	-0.09 ~ 0.15	-0.54 ~ 0.93
ν	-0.02 ~ 0.02	-0.72 ~ 1.08
Q_p	-0.42 ~ 0.36	-1.64 ~ 0.74

G-Thornton Case 6: m=5, n=4, σ_a=0.07, σ_b=0.09, $Cor \neq 0$		
para.	δ_{deep} %	δ %
$\widetilde{T}_{m02}^{-2}$	-0.39 ~ 0.25	-1.06 ~ 0.97
$\widetilde{T}_{m01}^{-1}$	-0.14 ~ 0.10	-0.36 ~ 0.94
ε	-0.24 ~ 0.11	-0.61 ~ 0.44
$\widetilde{T}_{-01}$	-0.06 ~ 0.10	-0.18 ~ 0.36
$\widetilde{T}_{-02}^{2}$	-0.09 ~ 0.15	-0.29 ~ 0.62
ν	-0.02 ~ 0.02	-0.63 ~ 1.70
Q_p	-0.42 ~ 0.36	-1.88 ~ 0.82

4. Conclusions

The coefficients of accurate approximate expressions for integral quantities and spectral width parameters for corrected versions of the generalized TMA and Thornton spectra in finite-depth water are provided for the 6 combinations of spectral parameter values. The relative error in each case is summarized. The results are presented in tables which may be very useful for a highly accurate estimation of the spectrum-integrated quantities.

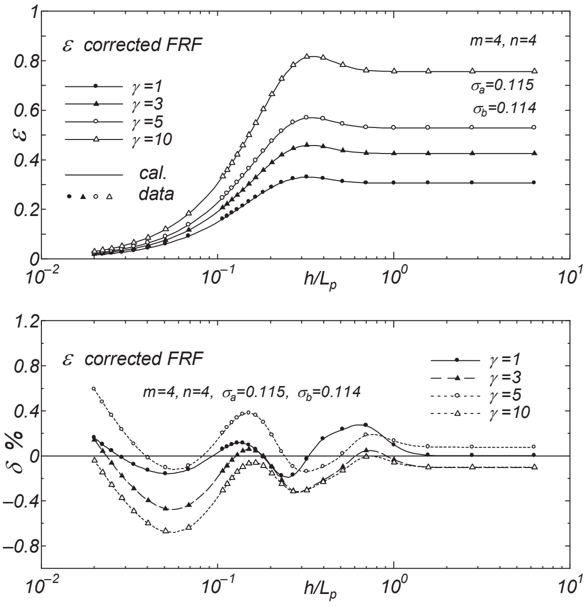

Figure 1 Comparison between approximate expression and data for dimensionless energy, and relative error on a whole range of h/L_p

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