Mem. Fac. Educ. Ehime Univ.,Nat. Sci., Vol. 15, No. 1 83~98 (1994)

PWM Amplifier and Its Application to Sinusoidal Wave Inverter

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(Received April 28, 1994)

ABSTRACT

This paper deals with both a power amplifier equipped with switched type puls-width-modulated (PWM) operation and its application to the compound method PWM sinusoidal wave inverters is proposed.

This PWM amplifier and inverters are clarified by means of the frequency spectrum distributions of the output PWM signals analyzed by double Fourier series expansions, and its spectrum distributions are shown by the experimental results.

We describe that a new switching method and compound method (PWM sinusoidal wave inverters) are available for obtaining good sinusoidal output signals and applicable to AC power sources for the devices, such as UPS, CVCF and the use of high frequency power inverters.

Key words: Power electronics, PWM amplifier, PWM sinusoidal wave inverter, Frequency spectrum distribution, double Fourier series expansion, switched modulation type PWM, compound method PWM, optimum PWM method, high frequency power inverter, Uniterruptible power supply, constant voltage constant frequency power source.

1. Introduction

Since pulse-width modulation (hereafter abbreviated as PWM) is a simplified method and yet is capable of providing high output power, various amplifiers for power circuits have been devised, and several practicable methods have already been realized.[(1)~(4)] Especially, when the input signals to PWM are limited to those having sinusoidal waveforms, it is usable as the sinusoidal waveform inverter utilizable in the recent power electronic technology fields, and so it is applicable to AC power sources for the devices such as UPS (uninterruptible power supply), CVCF (constant voltage constant frequency), and so on.[5][6][8]

In the present paper, the basic principles, the configuration methods, and characteristics of this PWM sinusoidal wave amplifier are presented, and particularly, due to the excellent characteristics of a switched modulation type PWM sinusoidal waveform amplifier, its utilization as inverter devices is described. And then a compound PWM sinusoidal inverter, or the theoretically extended method thereof, as an inverter adopting this optimum PWM method, are proposed; the characteristics of each method is clarified by means of double Fourier series expansions, computer simulations, and experiments on trial models; and their applications are described.

2. Basic configurations and operation principles of PWM amplifier

Figs. 1 and 2 show the basic configurations and the operating principles of PWM amplifier, respectively. The amplitude of the input signal $V_i(t)$ is compared with that of a

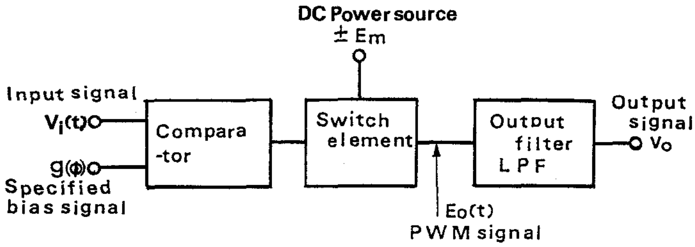

Fig. 1 PWM amplifier basic configuration.

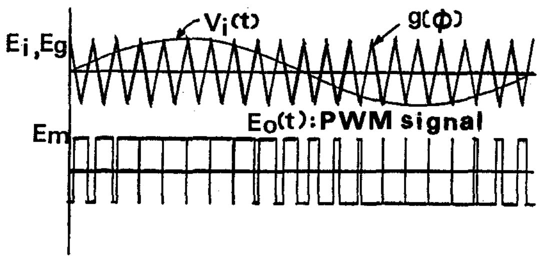

Fig. 2 Operation principle of both-edge PWM amplifier.

specific bias signal $g(\phi)$, and a DC power source ($\pm E_m$) is switched on and off at each time period T to engender the PWM signal waveform series $E_o(t)$ shown in Fig. 2(b). As this $E_o(t)$ is averaged by an output filter, the output signal $V_o(t)$, or the power amplified input signal $V_i(t)$, can be obtained.

Now, let the phase angles when the switch is turned on, and off, be represented by ϕ_P and ϕ_F, respectively, then the input-output relationship thereof is given by,

$$\frac{V_o}{V_i}=\frac{\overline{E_o}}{V_i}=\frac{1}{T}\int_{\phi_P}^{\phi_F}\frac{(\pm E_m)}{g(\phi)}\,d\phi=k \qquad (1)$$

where, k is amplification factor

To obtain a certain amplification factor, the following formula is obtained from (1).

$$g(\phi) = \pm\frac{1}{T}\int\frac{E_m}{k}\,d\phi \tag{2}$$

For a specific bias signal, an integrated waveform of the power source may be adopted.

Here, the PWM signals $E_o(t)$, when a single sinusoidal waveform (E_i sin $\omega_s t$) is input, assuming that the repetition angular frequency of the specific bias signal is ω_c ($=2\pi/T$), can be expanded generally by the following double Fourier series, and the frequency spectrum is obtained as follows.

$$E_o(t) = \sum_{m,\,n=-\infty}^{\infty}\sum C(m,\,n)e^{j(m\omega_s+n\omega_c)t} \tag{3}$$

where, the Fourier coefficients are given by,

$$C(m,\,n) = \frac{1}{(2\pi)^2}\int_0^{2\pi}\int_{\phi_P}^{\phi_F}(\pm E_m)e^{-j(m\omega_s+n\omega_c)t}d\omega_c t d\omega_s t \tag{4}$$

and the amplitudes of each frequency component can be obtained.[7]

Fig. 3 shows an example of the spectrum distribution of the PWM signals shown in Fig. 2 (a triangular wave having amplitude of E_g is adopted as the specific bias signal, under the condition of $M=E_i/E_g<1$), which is known as both-edge type PWM, and the magnitude represents the relative values of each harmonic wave component and the power source voltage. In the figure, "X" marks denote the theoretical values of each of those obtained from (4), and coincides completely with the measured values thereof.

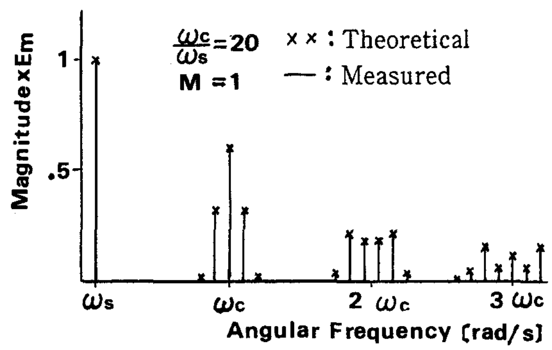

Fig. 3 Both-edge PWM frequency spectrum.

If $\omega_s \ll \omega_c$, than the input signals (ω_s component) are distributed in the low frequency region sufficiently separated from other components (we call these as 'unnecessary harmonic wave components'), and so they can be extracted by using a low pass filter (LPF) as a output circuit; consequently, an amplifier having an amplification factor $k=E_m/E_g$ with linear characteristics can be realized.

Now, assuming that the ratio (SN ratio) of the input signal components to the unnecessary harmonic wave components is equal to or larger than -50 dB, since the maximum angular frequency ω_{sm} of the input signal to this amplifier can be satisfied provided that ω_s is separable from $\omega_c - 4\omega_s$, the following formula (5) can be written.

$$\omega_{sm} = \omega_c/5 \tag{5}$$

A LPF, having a cut-off characteristic of 27 dB/oct or larger at the cut-off angular frequency of $\omega_c/5$, is needed for the output filter. In addition, in the case of the modulation system of the compound both-edge PWM type adopting positive and negative DC power source (double polarity), as shown in Fig. 4, the frequency spectrum of the unnecessary

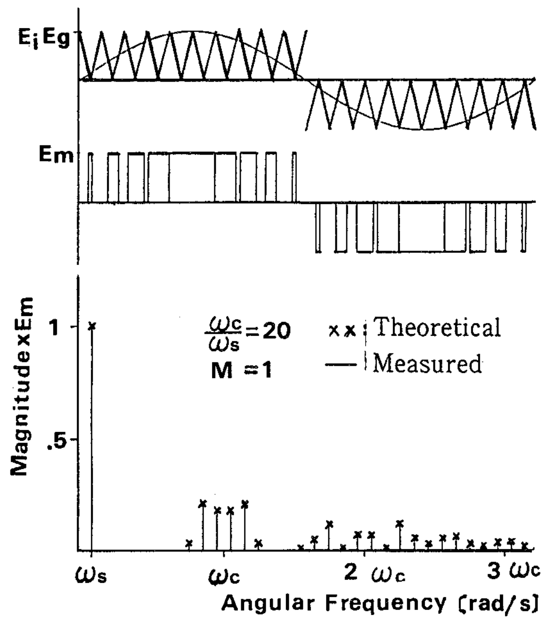

Fig. 4 Double-polar both-edge PWM operation and its frequency spectrum.

harmonic wave components includes only $2n\,\omega_c$ sideband components of the both-edge PWM (Fig. 3). Its maximum amplitude value diminishes, and the upper limit angular frequency of the input sinusoidal wave signals, capable of being amplified, is lowered.

In such a way, for the realization of the PWM sinusoidal wave amplifier, its inherent characteristics are determined according to the sorts of PWM being employed. For a preferable configuration, the following two items are generally considered as the requirements to be met, i.e., ① The realization of the amplification mechanism intended for the increase of the amplitude of the output sinusoidal wave; ② The enlargement of the dynamic range and the improvement of the accuracy of the output waveforms, both of which are to be achieved by the degeneracy of the unnecessary harmonic wave components included in the PWM signals. Especially, to meet the requirements of ②, contrivance in the modulation method is needed.

3. Switched modulation type PWM sinusoidal wave amplifier

Fig. 5 shows the frequency spectrum of the PWM signals from the sinusoidal wave amplifier employing the switched modulation type PWM method,[(7)(9)] contrived as the measures to meet the above-mentioned requirements. In the amplifier circuit, the modulation function is so performed that, at the summit points of the input sinusoidal waves, the switching over from the leading edge to the trailing edge, or vice versa, is carried out, and Fourier coefficients thereof are given by,

$$\boldsymbol{C}(m,n)\begin{cases} =0 & ;\ \text{for } m \text{ even} \\ =-j\dfrac{E_m}{mn\pi^2}\Big[-2(-1)^{\frac{m-1}{2}}+(-1)^n\Big\{m\pi \boldsymbol{J}_n(n\pi M)+(-1)^{\frac{m-1}{2}}\,2\boldsymbol{J}_0(n\pi M) & \\ \quad +(-1)^{\frac{m-1}{2}}\,4m^2\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^k}{m^2-4k^2}\boldsymbol{J}_{2k}(n\pi M)\Big\}\Big] & ;\ \text{for } m \text{ odd} \end{cases} \tag{6}$$

where, $M=E_i/E_g$, and $\boldsymbol{J}_m(X)$ is Bessel function of m-th order.

The frequency spectrum distributions of the unnecessary harmonic wave components

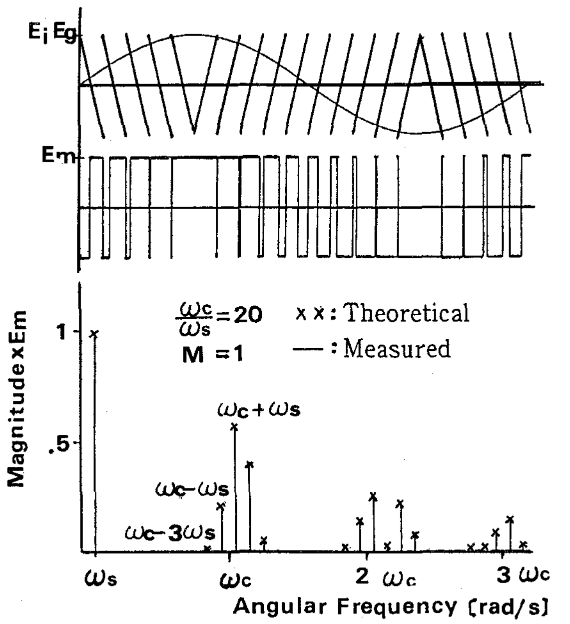

Fig. 5 Switched modulation PWM sinusoidal amplifier and its frequency spectrum.

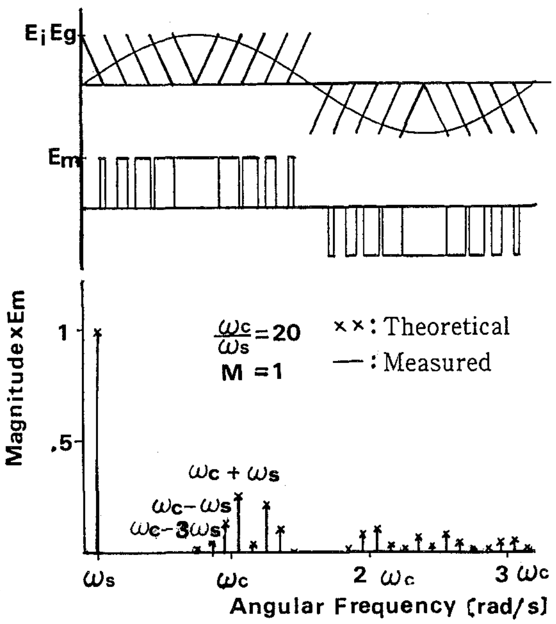

Fig. 6 2−step switched modulation PWM operation and its frequency spectrum.

are antisymmetric against $n\omega_c$. The theoretical and measured values coincide well each other, and thus the amplification of the sinusoidal wave signals up to $\omega_c/4$, by means of the output filter having characteristics comparable to those of the both–edge PWM amplifier, is made possible.

Fig. 6 shows the two stage switched type PWM sinusoidal wave amplifier in which the above–mentioned switched modulation method is employed in the both polarity PWM. In this case, since the values of Fourier coefficients $C(m, n)$ from (6) are halved, although the upper limits of the angular frequency of input signals are the same as the case shown in Fig. 5, the cut–off characteristic of LPF can be set to 16 dB/oct, or the half value of the case of Fig. 5, which facilitates the actual configuration of the output filter. If these operations are performed by employing high output power transistors (e.g., MOS FET, IGPT, etc.) as the switching elements for these types of amplifiers, each of them is utilizable as the inverter by which sinusoidal wave AC can be obtained from DC source. And they are applicable for AC power sources having excellent waveforms as well as for electric power controllers.

4. Compound PWM sinusoidal wave inverter[10]

4−1 Operating principle and circuit configuration

Fig. 7 shows the basic operating principle of the compound PWM sinusoidal converter, in which switching operation of the specific bias level of two stage switching type PWM method, as well as the level switching operation of DC power source voltage, are included.

This is the modulation method contrived for the purpose of further degeneration of the

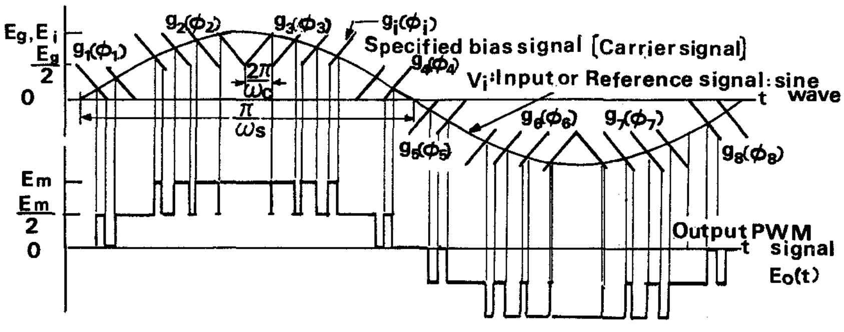

Fig. 7 Operation principles of compound PWM sinusoidal wave inverter.

unnecessary harmonic wave components included in the output PWM signals, and also of the extension of the upper limit of angular frequency for the input signal (hereafter it is called as the reference signal when it is concerned with an inverter).

As shown in the figure, eight sorts of specific bias signals (carrier signals), $g(\omega_c)$ are prepared in this modulation method, and they are used selectively and appropriately case-by-case corresponding to the sorts of reference signals. Its operation is controlled by the logical calculations concerning each of the detected signals as to the inclination, the polarity, and 1/2 level ($\pm E_i/2$) of the amplitude voltage of the reference signal. Namely, DC power source voltage E_m chopped off in the interval in which the amplitude voltage of the reference signal is greater than $E_i/2$, and DC power source voltage $E_m/2$ is chopped off in the interval in which the reference signal voltage take a value in the range $0 \sim E_s/2$. And completely same operations are performed also in the interval in which the amplitude voltage of the reference signal waves has negative polarity, to provide the output PWM signal $E_o(t)$ containing the reference signal information.

Fig. 8 shows an example of the basic circuit configuration for the compound PWM sinusoidal wave inverter intended to realize the above-mentioned operations.

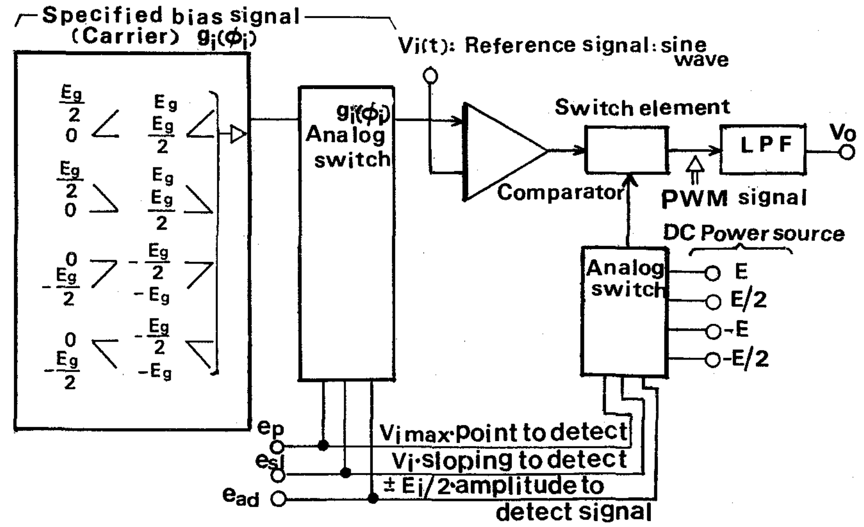

Fig. 8 Basic circuit construction of compound PWM sinusoidal wave inverter.

4−2 Analyses of PWM signal waveform series and the accuracy of the output waves

The Fourier coefficients of this compound PWM signal in the region $M \leq 1$ can be obtained by solving the equation (7),

$$C(m, n) = \frac{E_m}{2\pi^2}\left[\frac{1}{2}\int_0^{\sin^{-1}(\frac{1}{2M})}\int_{\varphi_1}^{2\pi} e^{-j(m\omega_s t + n\omega_c t)} d\omega_c t d\omega_s t \right.$$
$$+ \int_{\sin^{-1}(\frac{1}{2M})}^{\frac{\pi}{2}} \left\{\frac{1}{2}\int_0^{\varphi_2} e^{-j(m\omega_s t + n\omega_c t)} d\omega_c t\right.$$
$$\left. + \int_{\varphi_2}^{2\pi} e^{-j(m\omega_s t + n\omega_c t)} d\omega_c t\right\} d\omega_s t$$
$$+ \int_{\frac{\pi}{2}}^{\pi - \sin^{-1}(\frac{1}{2M})} \left\{\int_0^{\varphi_3} e^{-j(m\omega_s t + n\omega_c t)} d\omega_c t\right.$$
$$\left. + \frac{1}{2}\int_{\varphi_3}^{2\pi} e^{-j(m\omega_s t + n\omega_c t)} d\omega_c t\right\} d\omega_s t$$
$$\left. + \frac{1}{2}\int_{\pi - \sin^{-1}(\frac{1}{2M})}^{\pi}\int_0^{\varphi_4} e^{-j(m\omega_s t + n\omega_c t)} d\omega_c t d\omega_s t\right] \quad (7)$$

where, m is odd

and PWM output signal voltage $E_o(t)$ is given by the following formula.

$$E_o(t) = E_m M \sin \omega_s t$$
$$+ \sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} (-1)^m \frac{E_m}{(2m-1)n\pi^2}\left[(-1)^n\left\{(-1)^m(2m-1)\frac{\pi}{2} J_m(4n\pi M)\right.\right.$$
$$\left.\left. - J_o(4n\pi M) - 2(2m-1)^2\sum_{k=1}^{\infty} J_{2k}(4n\pi M)\frac{(-1)^k}{(2m-1)^2 - 4k^2}\right\} + 1\right]$$
$$\cdot \left\{\sin(2m-1)\omega_s t + n\omega_c t\right\} \quad (8)$$

For the case of M≥1, there exist intervals in which no crossing point come out in the amplitude comparison between the input signals and specific vias signals. The intervals in question are, by defining γ as $\gamma = \mathrm{Sin}^{-1}(1/M)$, represented by $\gamma \sim (\pi - \gamma)$, $(\pi + \gamma) \sim (2\pi - \gamma)$ [rad]. The Fourier coefficients for this case can be obtained similar to those in the case of M≤1, and PWM output signal voltage $E_o(t)$ is given by,

$$E_o(t) = \frac{2}{\pi} E_m\left[\cos\left\{\sin^{-1}\left(\frac{1}{M}\right)\right\} + M \sin^{-1}\left(\frac{1}{M}\right)\right]$$
$$+ \sum_{\substack{n=-\infty \\ m\neq 0}}^{\infty} \frac{2}{\pi} E_m M\left[\frac{1}{2m}\sin\left\{2m \sin^{-1}\left(\frac{1}{M}\right)\right\} + \frac{1}{2(m+1)}\sin\left\{2(m+1)\sin^{-1}\left(\frac{1}{M}\right)\right\}\right.$$
$$\left. + \cos\left\{(2m+1)\sin^{-1}\left(\frac{1}{M}\right)\right\}\right]\sin(2m+1)\omega_s t$$
$$- \sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} \frac{E_m}{(2m-1)n\pi^2}\left[\sin\left\{(2m+1)\sin^{-1}\left(\frac{1}{M}\right)\right\}\right.$$

$$- (2m-1)\ J_m(4n\pi M)\sin^{-1}\left(\frac{1}{M}\right)$$

$$- (2m-1) \sum_{\substack{k=-\infty \\ k \neq m}}^{\infty} J_k(4n\pi M)\ \frac{1}{2m-1-k}\ \sin\left\{(2m-1-k)\sin^{-1}\left(\frac{1}{M}\right)\right\}\Bigg]$$

$$\cdot \left\{\sin(2m-1)\,\omega_s t + n\omega_c t\right\} \qquad (9)$$

Fig. 9 shows, by graphical comparisons between theoretical and measured values, the frequency spectrum distribution of the output PWM signals obtainable by this compound modulation method, vis-a-vis the amplitude ratio M.

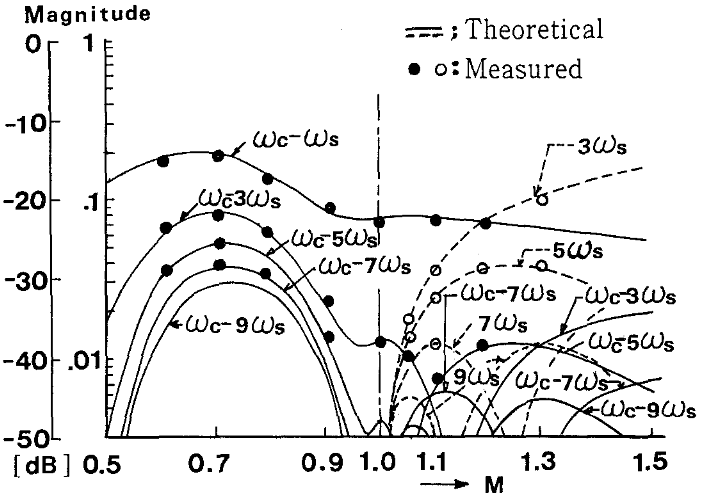

Fig. 9 Frequency spectrum distribution of the PWM signals obtainable by compound modulation method, visavis the amplitude retio M.

From the figure, it is seen that the harmonic wave components of $(\omega_c - n\omega_s)$ attain a minimum value in the neighborhood of $M=1$, then as the M's value decreases the former increases to attain its maximum value in the neighborhood of $M=0.7$, and at $M=0.5$ it takes a second lowest value against the minimum one in the case of $M=1$. In the region of $M>1$, the harmonic wave component decreases once temporarily as the M's value increase to take such a value as -46dB or lower against signal component. After which it tends to increase again, and the harmonic wave components of the reference signal, nonexisting in the region of $M<1$, is engendered. Among them, $3\omega_s$ component increases rapidly in accordance with the increase of M's value, and other higher harmonic wave components such as $5\omega_s$, $7\omega_s$, $\cdots$ also take larger values than $(\omega_c - n\omega_s)$ components. Since these higher harmonic wave components are distributed in the frequency region close to that of the input signals, separation of the formers from the reference signal components by means of LPF becomes difficult, resulting in a cause of the distortion in the inverter's output waveforms. Therefore, in this compound modulation method, by setting the M's value in the range $0.95 \leq M \leq 1.02$, and also by setting the cut-off angular frequency of the LPF to $\omega_c/4$ and its steepness of attenuation characteristic to 14 dB/oct, an inverter having a favorable output waveform accuracy can be obtained. However, in this modulation method, a problem of the errors in the switched modulation at the maximum amplitude of the reference sinusoidal wave also comes out.

Now, by letting the delay angle of the switched modulation point from the summit point of the reference sinusoidal wave signal, be ϕ (rad), then the Fourier coefficients, consisting of five sorts of components concerning the output PWM waveform series, for the region of $M \leq 1$, are given by,

(1) DC component

$$\boldsymbol{C}(0,\ 0)=0 \tag{10}$$

(2) Signal component

$$\boldsymbol{C}(0,\ 0)=-j\,\frac{E_m}{2}M \tag{11}$$

(3) Higher harmonic wave component of reference signal

$$\boldsymbol{C}(m,\ 0)=0 \tag{12}$$

(4) Higher harmonic wave component of the specific bias signal

$$\boldsymbol{C}(0,\ n)=0 \tag{13}$$

(5) Sum and difference components between the higher harmonic wave of specific bias signal and the signal's higher harmonic wave

$$\begin{aligned}\boldsymbol{C}(m,\ n)=\frac{E_m}{2n\pi^2}\Bigg[&\frac{1}{m}(-1)^{\frac{m-1}{2}}\sin(m\phi)\\ &+\sum_{k=-\infty}^{\infty}\boldsymbol{J}_{2k}(4n\pi M)\frac{1}{m-2k}(-1)^{\frac{m-2k-1}{2}}\sin(m-2k)\phi\\ &+j\,\frac{1}{m}(-1)^{\frac{m-1}{2}}\cos(m\phi)-j\,\frac{\pi}{2}\,\boldsymbol{J}_m(4n\pi M)\\ &-j\sum_{k=-\infty}^{\infty}\boldsymbol{J}_{2k}(4n\pi M)\frac{1}{m-2k}(-1)^{\frac{m-2k-1}{2}}\cos(m-2k)\phi\Bigg]\end{aligned} \tag{14}$$

where, m is odd

Fig. 10 shows the variation of the amplitude values due to the deviated switching angle ϕ, of the major unnecessary harmonic wave components, at $M=1$, and it is seen that the frequency spectrum remains completely invariable if ϕ takes a value in the range, $\pm 13°$. Fig. 11(b) shows an example of this frequency spectrum distribution, and Fig. 11(a) shows an example of the operating waveforms of an inverter fabricated on experimental basis. V_o is an example of the output waveform demodulated by a LPF having 12 dB/oct characteristic. Since it is understood theoretically that if ϕ is $\phi=\pm 26°$, the maximum value of the unnecessary higher harmonic wave components decreases to 12.7% (−18 dB) of the reference signal components, it can be made possible to output sinusoidal wave signals in the frequency range up to $\omega_c/4$ by using a LPF having cut-off characteristic of 14 dB/oct as

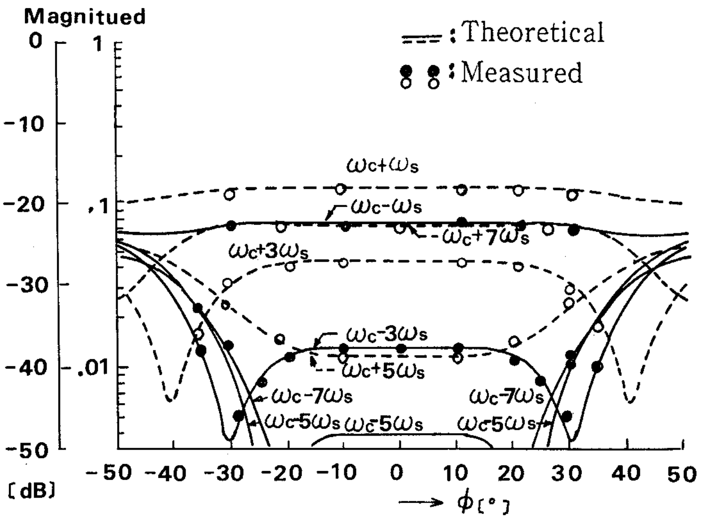

Fig. 10 Variation of the amplitude values due to the deviated switching angle ϕ, of the major harmonic components, at M=1.

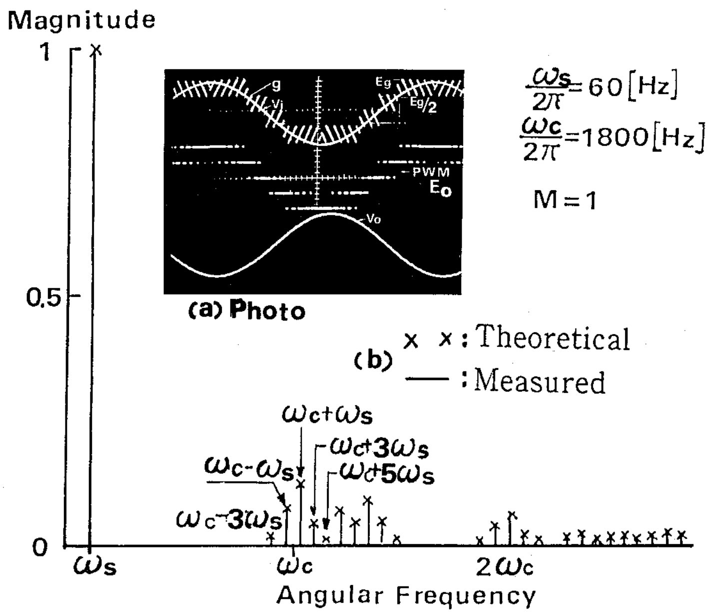

Fig. 11 Experimental results due to compound PWM sinusoidal wave inverter.

(a) Photo: Upper is specified bias(g) and reference signal(V_i) waveform, middle is PWM signal(E_o) waveform, lower is output voltage(V_o) waveform.
$\omega_s/2\pi=60$ [Hz], $\omega_c/2\pi=1200$ [Hz].
$\omega_{cut}/2\pi=300$ [Hz], 12dB/oct.
(b) Frequency spectrum of PWM signal (M=1).

an output filter. In addition, the distortion factor d, or an index representing the accuracy of this output signal waveform, is given by,[(11)]

$$d=\frac{\sqrt{\sum_{m=2}^{\infty}\{\varepsilon C(m,0)\}^2+\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty}\{\varepsilon C(m,n)\}^2}}{|\varepsilon C(1,0)|}\times 100[\%] \tag{15}$$

Fig. 12 is represented by taking the variation of cut-off frequency to the variation of amplitude ratio M as a parameter when ε is a attenuation constant given by the function of frequency and the attenuation characteristics of output filter is set to 12 [dB/oct].

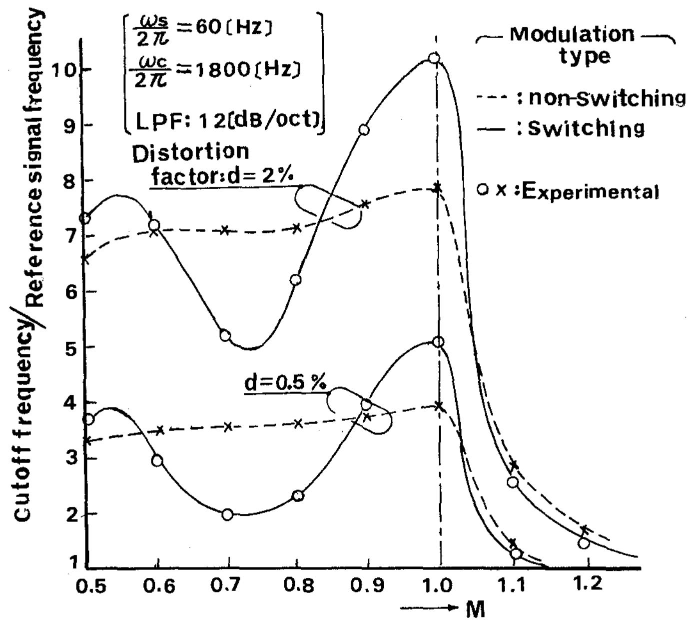

Fig. 12 Variation of cut-off frequency to the variation of amplitude ratio M as a parameter when distortion factor of output signal waveform.

From the figure, it is seen that, so long as M's value is set close to 1 (provided that $M \leq 1$), to obtain the same accuracy of waveform, the cut-off frequency of the output filter can be set to higher value for this compound PWM method than any other type of inverters not employing modulation technique, from which it can be said that the design requirements of the output filter may be alleviated. In other words, the waveform accuracy of this PWM inverter is excellent, and for the case in which the frequency ratio of the reference signal to specific bias signal is not sufficiently high, it is an utilizable inverter to be used in UPS (uninterruptible power supply)[11] for the devices such as a high frequency inverter, OA equipments and so on requiring good waveform accuracy.

4−3 Conciderations on optimum working compound PWM[12][13]

With respect to the frequency spectrum distribution of the compound PWM output signal, shown in Fig. 11(a), it would be an ideal case if only the sum ($m\omega_c+n\omega_s$, where m and n denote even and odd numbers, respectively), of the higher harmonic wave components ($m\omega_c$) of specific bias signals and those ($n\omega_s$) of the reference signals are present in the output signals, and if the difference components are furthermore attenuated. Fig. 13

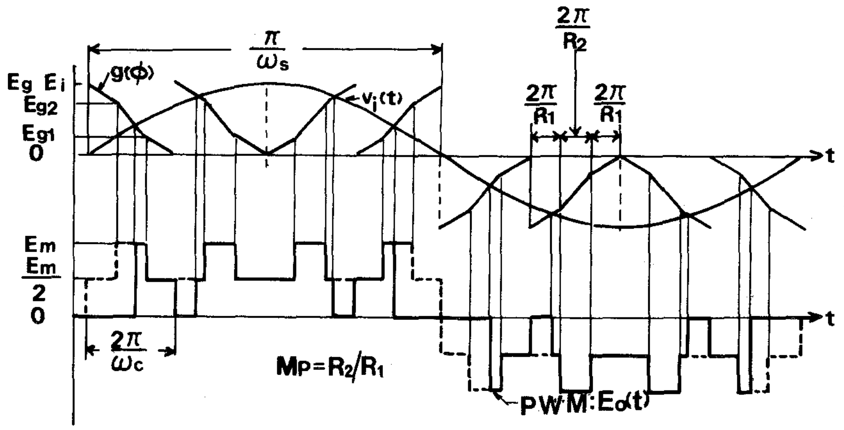

Fig. 13 Operation principles of optimum working compound PWM sinusoidal inverter.

shows the generation principle of the optimum working compound PWM waveform series which is one of the solving measures against the above-mentioned requirements.

The principle consists in, firstly adopting a sinusoidal wave as the reference signal, and a polygonal wave (the inclination ratios of three segments occupying three intervals R_1, R_2, and R_i being set to 1 : 2 : 1) playing a role of an approximated inverse cosine wave having two sorts of inclinations as the specific bias signal, then the conductive phase angles of the switching elements are controlled by adopting as control signals the comparison crossing point of the above-mentioned two signals. And then, seeking the ratio of R_1 to R_2, by which the frequency spectrum distribution of the PWM signals obtained in the above-mentioned process is a favorable one (which is called the power source voltage interval ratio M_p), determining the shape of the polygonal line of the specific bias waveform, and thus an optimum PWM waveform series is engendered.

The Fourier coefficients of this PWM signal, when $M \leq 1$, is given by,

(1) DC component

$$\boldsymbol{C}(0,\ 0)=0 \tag{16}$$

(2) Signal component

$$\boldsymbol{C}(1,\ 0)=-j\ \frac{R_1+R_2}{2\pi^2}\left\{M \sin^{-1}\frac{1}{M}+\cos\left(m \sin^{-1}\frac{1}{M}\right)\right\} \tag{17}$$

(3) Higher harmonic wave component of reference signal

$$\boldsymbol{C}(m,\ 0)=\begin{cases} 0 & ;\ \text{for } m \text{ even} \\ j\ \dfrac{R_1+R_2}{2\pi^2}\left[M\left\{\dfrac{\sin(m+1)\sin^{-1}\frac{1}{M}}{m+1}-\dfrac{\sin(m-1)\sin^{-1}\frac{1}{M}}{m-1}\right\}\right. & \\ \left.-\dfrac{2}{m}\cos\left(m \sin^{-1}\dfrac{1}{M}\right)\right] & ;\ \text{for } m \text{ odd } (m \neq 1) \end{cases} \tag{18}$$

(4) Higher harmonic wave component of the specific bias signal

$$\boldsymbol{C}(0,\ n)=0 \tag{19}$$

(5) Sum and difference components between the higher harmonic wave of specific bias signal and the signal's higher harmonic wave

$$\boldsymbol{C}(m,\ n)=\begin{cases} 0 & ;\ \text{for } m \text{ even} \\ j\,\dfrac{E_m}{2n\pi^2}\Big[\dfrac{1}{m}\Big\{\sin\{m\ \sin^{-1}\Big(\dfrac{R_1+2R_2}{2(R_1+R_2)M}\Big)+nR_1\} & ;\ \text{for } m \text{ odd} \\ -\sin\{m\ \sin^{-1}\Big(\dfrac{R_1}{2(R_1+R_2)M}\Big)-nR_1\}+\sin\Big(m\ \sin^{-1}\dfrac{1}{M}\Big)\Big\} \end{cases}$$

$$-\sin^{-1}\Big(\frac{R_1}{2(R_1+R_2)M}\Big)\boldsymbol{J}_m\{2(R_1+R_2)nM\}$$

$$-\sum_{\substack{k=-\infty\\k\neq m}}^{\infty}\boldsymbol{J}_k\{2(R_1+R_2)nM\}\frac{1}{m-k}\ \sin\{(m-k)\sin^{-1}\Big(\frac{R_1}{2(R_1+R_2)M}\Big)\}$$

$$-2\ \cos\Big(\frac{n}{2}\ R_1\Big)\Big\{\{\sin^{-1}\Big(\frac{R_1+2R_2}{2(R_1+R_2)M}\Big)-\sin^{-1}\Big(\frac{R_1}{2(R_1+R_2)M}\Big)\}$$

$$\cdot\boldsymbol{J}_m\{(R_1+R_2)nM\}+\sum_{\substack{k=-\infty\\k\neq m}}^{\infty}\boldsymbol{J}_k\{(R_1+R_2)nM\}\frac{1}{m-k}$$

$$\cdot\sin\{(m-k)\sin^{-1}\Big(\frac{R_1+2R_2}{2(R_1+R_2)M}\Big)\}-\sin\{(m-k)\sin^{-1}\Big(\frac{R_1}{2(R_1+R_2)M}\Big)\}\Big\}$$

$$+2\ \sin\Big(\frac{n}{2}\ R_1\Big)\sum_{\substack{k=-\infty\\k\neq m}}^{\infty}\boldsymbol{J}_k\{(R_1+R_2)nM\}\frac{1}{m-k}$$

$$\cdot\ \Big\{\cos\{(m-k)\sin^{-1}\Big(\frac{R_1+2R_2}{2(R_1+R_2)M}\Big)\}-\cos\{(m-k)\sin^{-1}\Big(\frac{R_1}{2(R_1+R_2)M}\Big)\}\Big\}$$

$$+\cos(nR_2)\ \Big\{\{\sin^{-1}\Big(\frac{R_1+2R_2}{2(R_1+R_2)M}\Big)-\sin^{-1}\ \frac{1}{M}\}\boldsymbol{J}_m\{2(R_1+R_2)nM\}$$

$$+\sum_{k=-\infty}^{\infty}\boldsymbol{J}_k\{2(R_1+R_2)nM\}\frac{1}{m-k}\ \sin\{(m-k)\sin^{-1}\Big(\frac{R_1+2R_2}{2(R_1+R_2)M}\Big)\}$$

$$-\sin\{(m-k)\sin^{-1}\ \frac{1}{M}\}\Big\}$$

$$+\sin(nR_2)\sum_{\substack{k=-\infty\\k\neq m}}^{\infty}\boldsymbol{J}_k\{2(R_1+R_2)nM\}\frac{1}{m-k}$$

$$\cdot\ \Big\{\cos\{(m-k)\sin^{-1}\Big(\frac{R_1+2R_2}{2(R_1+R_2)M}\Big)\}-\cos\{(m-k)\sin^{-1}\ \frac{1}{M}\}\Big\}\Big] \tag{20}$$

Fig. 14 shows the amplitude variation of the major harmonic components against M_p (|Amplitude values of each harmonic wave components|/|Input signal components|) at the point $M=1$. Against the value of M_p the component $\omega_c-3\omega_s$ hardly varies, while the other components varies a good deal, and each of the latters takes minimum values at respective two points, and when $M_p=3.72$, the average values of each component takes their respective minimum values. However, when the value of M_p lies in the range 3.72~4.52, components $\omega_c-5\omega_s$ and $\omega_c-7\omega_s$ do not vary too much and component $\omega_c-\omega_s$ takes a minimum value, which leads to a practicable selection of the components' values against the range of M_p.

Fig. 15(a) shows an example of each of the operating waveforms and the output sinusoidal waveforms as to the optimum working compound PWM inverter fabricated on experimental basis, and Fig. 15(b) shows an example of the frequency spectrum distribution

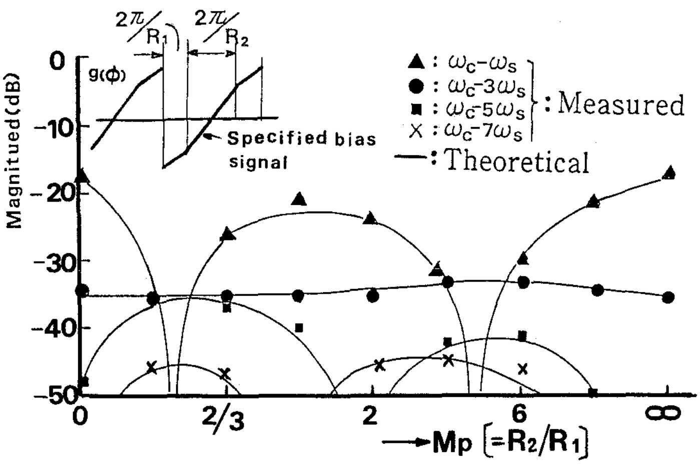

Fig. 14 Main harmonic components are convorted in to composite index of supply voltage M_p.

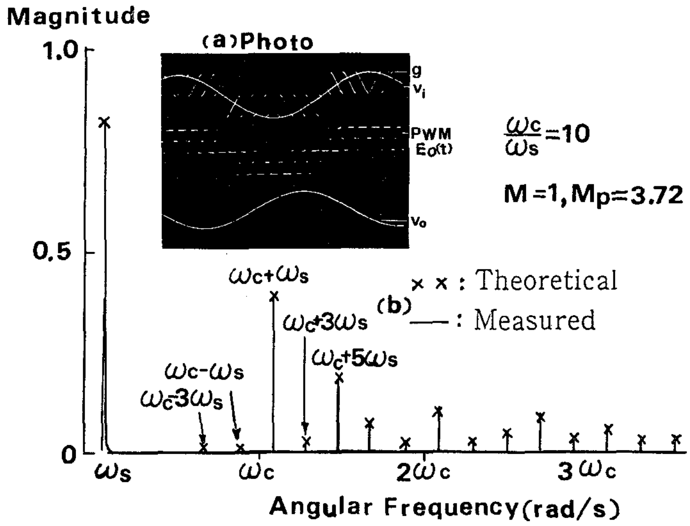

Fig. 15 Experimental result due to optimum working compound sinusoidal PWM inverter.

(a) Photo: Upper is specified bias (g) and reference signal (V_i) waveform, middle is PWM signal (E_o), lower is output voltage (V_o) waveform.
$\omega_s/2\pi=60$ [Hz], $\omega_c/2\pi=900$ [Hz], $\omega_{cut}/2\pi=100$ [Hz], 12dB/oct.
(b) Frequency spectrum of bast compounded PWM signal (M=1).

measured thereof. "X" marks in the figure denote theoretical values, and since they coincide well with the measured values, the validity of our theory is confirmed. The measured values of each component at the optimum pattern ($M=1$, $M_p=3.72$), of the output signals against the corresponding reference signal components take such values as $\omega_c+\omega_s=-6.7$ dB, $\omega_c-\omega_s=-40.0$ dB, $\omega_c-3\omega_s=-36.3$ dB, and $\omega_c-5\omega_s=-45.1$ dB. It is seen that component $\omega_c-3\omega_s$ is somewhat large, but all of the differences of harmonic waves' components between the other specific bias signals and the reference signals are -40 dB or lower. From these facts, so long as M takes a value close to 1 ($0.9<M<1.05$), since the upper limit frequency of the reference signal is confined by the component's difference $[\omega_c-3\omega_s]$ between the harmonic wave of specific bias signal and the odd number higher harmonic waves, more improvement of the dynamic range, compared with the case of the compound PWM sinusoidal wave inverter shown in Fig. 7, can be achieved, leading to the applicability to higher frequency devices.

5. Conclusion

Basic principles and representative configuration methods of PWM amplifiers have been presented, their frequency spectra were obtained by expanding the PWM signals, on the occasion when the input signals are confined to sinusoidal waves, in double Fourier series, and the characteristics thereof were described. Especially, the two stage PWM sinusoidal wave amplifier, in which the pulse width is switched over from the leading edge to trailing edge, or to leading edge modulation, makes possible the lowering of the amplitudes of unnecessary harmonic wave components, and also the enlargement of the dynamic range (the upper limit value of input sinusoidal wave frequency), which proved the applicability of the present amplifier as inverter.

Next, we proposed a compound PWM sinusoidal wave inverter which is a logically extended version of the two stage PWM sinusoidal wave amplifier employing this switched modulation method. From the studies on theoretical analyses and the experimental results based on the devices prepared by trial fabrication, it was clarified that the basic characteristics of the new inverter provide sinusoidal wave output having very low distortion rate, and that it is effectively applicable to CVCF (constant voltage-constant frequency) device and also to UPS (uninterruptible power supply) for OA appliances.

Moreover, we have replaced the specific bias signal waves in this compound PWM method with a polygonal line waveforms (using a sawtooth waveforms with differently inclined segments) having two sorts of inclinations approximated to inverse cosine function, and made propositions and studies on the models made by trial fabrication concerning the optimum compound PWM method in which upwardly convex type DC power source voltage is adopted. From these studies it was shown that, since the upper limit frequency of the reference sinusoidal waveform signal is confined by the difference between the harmonic components of the secondary specific bias signal (carrier signal) and the odd number higher harmonic components of the reference signal to delimit the upper frequency at $\omega_c - 3\omega_s$, which facilitates the configuration of the output filter (LPF), and that it is applicable to inverters for high frequency circuit devices in which the frequency ratio of the reference signal to carrier signal cannot be set to a large number. Moreover, by means of the sinusoidal wave approximation of DC power source voltage using this optimum compound PWM method, it is possible to realize as the unnecessary harmonic wave components of a certain PWM signal such that the former comprise only the components' sum of the two signals, i.e., odd number harmonic waves of the reference signal and the harmonic waves of carrier signal. However, as to this proposed new scheme, it will later be reported in another paper, together with VVVF (variable voltage, variable frequency) method in which the reference signal itself varies, and also with the development of an inverter in which further multiplexing techniques are applied to PWM.

Finally, the authors express sincere gratitude to Prof. Shukichi Kaku (Osaka Institute of Technology) who participated in the discussion of the present research and gave us valuable advices.

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